***Solution*** ***Section* 2.5 – Numerical Integration**

***Exercise***

Find the Midpoint Rule approximations to: 

***Solution***



























































***Exercise***

Find the Midpoint Rule approximations to: 

***Solution***









































***Exercise***

Find the Midpoint Rule approximations to: 

***Solution***































***Exercise***

Find the Midpoint Rule approximations to: 

***Solution***





































***Exercise***

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of  by (***a***) the Trapezoid Rule and (***b***) Simpson’s Rule. 

***Solution***

1.  





|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | 1 | 1 | 1 | 1 |
|  |  | 2 | 2 | 4 |
|  | 2 | 3 | 2 | 6 |
|  |  | 4 | 2 | 8 |
|  | 3 | 5 | 1 | 5 |
|  |  |  |  | **24** |

















|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | 1 | 1 | 1 | 1 |
|  |  | 2 | 4 | 8 |
|  | 2 | 3 | 2 | 6 |
|  |  | 4 | 4 | 16 |
|  | 3 | 5 | 1 | 5 |
|  |  |  |  | **36** |





1.  



























***Exercise***

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | −1 | 2 | 1 | 2 |
|  |  |  | 2 |  |
|  | 0 | 1 | 2 | 2 |
|  |  |  | 2 |  |
|  | 1 | 2 | 1 | 2 |
|  |  |  |  | **11** |

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of  by (***a***) the Trapezoid Rule and (***b***) Simpson’s Rule. 

***Solution***

1. 



































|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  |  |  |  |  |
|  | −1 | 2 | 1 | 2 |
|  |  |  | 4 | 5 |
|  | 0 | 1 | 2 | 2 |
|  |  |  | 4 | 5 |
|  | 1 | 2 | 1 | 2 |
|  |  |  |  | **16** |







1. 





























***Exercise***

Estimate the minimum number of subintervals to approximate the integrals with an error of magnitude of  by (***a***) the Trapezoid Rule and (***b***) Simpson’s Rule. 

***Solution***

1.  



































The percentage error: 



1.  

























The percentage error: 



***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

|  |  |  |  |
| --- | --- | --- | --- |
| *n* |  |  |  |
| 0 | 0.0000000000 | 0.0000000000 | 0.0000000000 |
| 1 | 0.1666666667 | 0.5000000000 | 1.0000000000 |
| 2 | 0.3333333333 | 0.8660254000 | 1.7320508000 |
| 3 | 0.5000000000 | 1.0000000000 | 2.0000000000 |
| 4 | 0.6666666667 | 0.8660254000 | 1.7320508000 |
| 5 | 0.8333333333 | 0.5000000000 | 1.0000000000 |
| 6 | 1.0000000000 | 0.0000000000 | 0.0000000000 |

*Trapezoid Rule* approximation ≈ 0.62200847

***Simpson's Rule*** Method

|  |  |  |  |
| --- | --- | --- | --- |
| *n* |  |  |  |
| 0 | 0.0000000000 | 0.0000000000 | 0.0000000000 |
| 1 | 0.1666666667 | 0.5000000000 | 2.0000000000 |
| 2 | 0.3333333333 | 0.8660254000 | 1.7320508000 |
| 3 | 0.5000000000 | 1.0000000000 | 2.0000000000 |
| 4 | 0.6666666667 | 0.8660254000 | 1.7320508000 |
| 5 | 0.8333333333 | 0.5000000000 | 1.0000000000 |
| 6 | 1.0000000000 | 0.0000000000 | 0.0000000000 |

***Simpson's Rule*** approximation ≈ 0.63689453

***Exact Trapezoid Simpson***

--------------------------------------------------------------

Value: 0.63661977 0.62200847 0.63689453

--------------------------------------------------------------

Error: 2.2951 % 0.0432 %

--------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations to and error to 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.1250000000 0.8824969000 1.7649938000

2 0.2500000000 0.7788007800 1.5576015600

3 0.3750000000 0.6872892800 1.3745785600

4 0.5000000000 0.6065306600 1.2130613200

5 0.6250000000 0.5352614300 1.0705228600

6 0.7500000000 0.4723665500 0.9447331000

7 0.8750000000 0.4168620200 0.8337240400

8 1.0000000000 0.3678794400 0.3678794400

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.63294342

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.1250000000 0.8824969000 3.5299876000

2 0.2500000000 0.7788007800 1.5576015600

3 0.3750000000 0.6872892800 2.7491571200

4 0.5000000000 0.6065306600 1.2130613200

5 0.6250000000 0.5352614300 2.1410457200

6 0.7500000000 0.4723665500 0.9447331000

7 0.8750000000 0.4168620200 1.6674480800

8 1.0000000000 0.3678794400 0.3678794400

-------------------------------------------------------------------

***Simpson's Rule*** approximation ≈ 0.63212141

***Exact Trapezoid Simpson***

--------------------------------------------------------------

Value: 0.63212056 0.63294342 0.63212141

--------------------------------------------------------------

Error: 0.1302 % 0.0001 %

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error to:



***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 1.5000000000 3.7500000000 7.5000000000

2 2.0000000000 8.0000000000 16.0000000000

3 2.5000000000 13.7500000000 27.5000000000

4 3.0000000000 21.0000000000 42.0000000000

5 3.5000000000 29.7500000000 59.5000000000

6 4.0000000000 40.0000000000 80.0000000000

7 4.5000000000 51.7500000000 103.500000000

8 5.0000000000 65.0000000000 65.0000000000

---------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 100.50000000

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 1.5000000000 3.7500000000 15.0000000000

2 2.0000000000 8.0000000000 16.0000000000

3 2.5000000000 13.7500000000 55.0000000000

4 3.0000000000 21.0000000000 42.0000000000

5 3.5000000000 29.7500000000 119.000000000

6 4.0000000000 40.0000000000 80.0000000000

7 4.5000000000 51.7500000000 207.000000000

8 5.0000000000 65.0000000000 65.0000000000

---------------------------------------------------------------------

***Simpson's Rule*** approximation ≈ 100.00000000

***Exact Trapezoid Simpson***

--------------------------------------------------------------

Value: 100.000000 100.500000 100.00000000

---------------------------------------------------------------

Error: 0.5000% 0.0000 %

---------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.0981747704 0.5852709700 1.1705419400

2 0.1963495408 1.1480503000 2.2961006000

3 0.2945243113 1.6667107000 3.3334214000

4 0.3926990817 2.1213203400 4.2426406800

5 0.4908738521 2.4944088400 4.9888176800

6 0.5890486225 2.7716386000 5.5432772000

7 0.6872233930 2.9423558400 5.8847116800

8 0.7853981634 3.0000000000 3.0000000000

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 1.49517776

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.0981747704 0.5852709700 2.3410838800

2 0.1963495408 1.1480503000 2.2961006000

3 0.2945243113 1.6667107000 6.6668428000

4 0.3926990817 2.1213203400 4.2426406800

5 0.4908738521 2.4944088400 9.9776353600

6 0.5890486225 2.7716386000 5.5432772000

7 0.6872233930 2.9423558400 11.7694233600

8 0.7853981634 3.0000000000 3.0000000000

--------------------------------------------------------------------

***Simpson's Rule*** approximation ≈ 1.50001244

***Exact Trapezoid Simpson***

-----------------------------------------------------------

Value: 1.500000 1.49517776 1.50001244

------------------------------------------------------------

Error: 0.3215 % 0.0008 %

------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 1.0000000000 0.1353352800 0.2706705600

2 2.0000000000 0.0183156400 0.0366312800

3 3.0000000000 0.0024787500 0.0049575000

4 4.0000000000 0.0003354600 0.0006709200

5 5.0000000000 0.0000454000 0.0000908000

6 6.0000000000 0.0000061400 0.0000122800

7 7.0000000000 0.0000008300 0.0000016600

8 8.0000000000 0.0000001100 0.0000001100

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.65651755

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 1.0000000000 0.1353352800 0.5413411200

2 2.0000000000 0.0183156400 0.0366312800

3 3.0000000000 0.0024787500 0.0099150000

4 4.0000000000 0.0003354600 0.0006709200

5 5.0000000000 0.0000454000 0.0001816000

6 6.0000000000 0.0000061400 0.0000122800

7 7.0000000000 0.0000008300 0.0000033200

8 8.0000000000 0.0000001100 0.0000001100

-------------------------------------------------------------------

***Simpson's Rule*** approximation ≈ 0.52958521

***Exact Trapezoid Simpson***

-----------------------------------------------------------

Value: 0.49999994 0.65651755 0.52958521

------------------------------------------------------------

Error: 31.3035 % 5.9171 %

-----------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 −1.0000000000 1.4142135600 1.4142135600

1 −0.7500000000 1.2500000000 2.5000000000

2 −0.5000000000 1.1180339900 2.2360679800

3 −0.2500000000 1.0307764100 2.0615528200

4 0.0000000000 1.0000000000 2.0000000000

5 0.2500000000 1.0307764100 2.0615528200

6 0.5000000000 1.1180339900 2.2360679800

7 0.7500000000 1.2500000000 2.5000000000

8 1.0000000000 1.4142135600 1.4142135600

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 2.30295859

***Simpson's Rule*** Method

*n *  

0 −1.0000000000 1.4142135600 1.4142135600

1 −0.7500000000 1.2500000000 5.0000000000

2 −0.5000000000 1.1180339900 2.2360679800

3 −0.2500000000 1.0307764100 4.1231056400

4 0.0000000000 1.0000000000 2.0000000000

5 0.2500000000 1.0307764100 4.1231056400

6 0.5000000000 1.1180339900 2.2360679800

7 0.7500000000 1.2500000000 5.0000000000

8 1.0000000000 1.4142135600 1.4142135600

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 2.29556453

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 2.29558715 2.30295859 2.29556453

----------------------------------------------------------------

Error: 0.3211 % 0.0010 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1250000000 0.0156243642 0.0312487284

2 0.2500000000 0.0624593178 0.1249186357

3 0.3750000000 0.1401619723 0.2803239447

4 0.5000000000 0.2474039593 0.2474039593

------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.0427434543

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1250000000 0.0156243642 0.0624974569

2 0.2500000000 0.0624593178 0.1249186357

3 0.3750000000 0.1401619723 0.5606478894

4 0.5000000000 0.2474039593 0.2474039593

---------------------------------------------------------------

*Simpson Rule* approximation ≈ 0.0414778309

***Exact Trapezoid Simpson***

------------------------------------------------------------------------

Value: 0.0414810243 0.0427434543 0.0414778309

------------------------------------------------------------------------

Error: 3.04339% 0.00770 %

------------------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 1.5708000000 0.6366182800 0.6366182800

1 1.8323333333 0.5271932200 1.0543864400

2 2.0938666667 0.4137271600 0.8274543200

3 2.3554000000 0.3004450800 0.6008901600

4 2.6169333333 0.1914141900 0.3828283800

5 2.8784666667 0.0903606800 0.1807213600

6 3.1400000000 0.0005072100 0.0005072100

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.48166674

***Simpson's Rule*** Method

*n *  

0 1.5708000000 0.6366182800 0.6366182800

1 1.8323333333 0.5271932200 2.1087728800

2 2.0938666667 0.4137271600 0.8274543200

3 2.3554000000 0.3004450800 1.2017803200

4 2.6169333333 0.1914141900 0.3828283800

5 2.8784666667 0.0903606800 0.3614427200

6 3.1400000000 0.0005072100 0.0005072100

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 0.48116938

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 0.48117214 0.48166674 0.48116938

----------------------------------------------------------------

Error: 0.1028 % 0.0006 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1308996939 0.0172332716 0.0344665433

2 0.2617993878 0.0701489345 0.1402978691

3 0.3926990817 0.1626612856 0.3253225711

4 0.5235987756 0.3022998940 0.6045997881

5 0.6544984695 0.5022143392 1.0044286785

6 0.7853981634 0.7853981634 0.7853981634

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.1894454730

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1308996939 0.0172332716 0.0689330865

2 0.2617993878 0.0701489345 0.1402978691

3 0.3926990817 0.1626612856 0.6506451423

4 0.5235987756 0.3022998940 0.6045997881

5 0.6544984695 0.5022143392 2.0088573569

6 0.7853981634 0.7853981634 0.7853981634

------------------------------------------------------------------

*Simpson Rule* approximation ≈ 0.1858222125

***Exact Trapezoid Simpson***

-----------------------------------------------------------------------

Value: 0.1857845357 0.1894454730 0.1858222125

-----------------------------------------------------------------------

Error: 1.97053% 0.02028 %

-----------------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.1000000000 0.9900498337 1.9800996675

2 0.2000000000 0.9607894392 1.9215788783

3 0.3000000000 0.9139311853 1.8278623705

4 0.4000000000 0.8521437890 1.7042875779

5 0.5000000000 0.7788007831 1.5576015661

6 0.6000000000 0.6976763261 1.3953526521

7 0.7000000000 0.6126263942 1.2252527884

8 0.8000000000 0.5272924240 1.0545848481

9 0.9000000000 0.4448580662 0.8897161324

10 1.0000000000 0.3678794412 0.3678794412

-----------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.7462107961

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.1000000000 0.9900498337 3.9601993350

2 0.2000000000 0.9607894392 1.9215788783

3 0.3000000000 0.9139311853 3.6557247411

4 0.4000000000 0.8521437890 1.7042875779

5 0.5000000000 0.7788007831 3.1152031323

6 0.6000000000 0.6976763261 1.3953526521

7 0.7000000000 0.6126263942 2.4505055767

8 0.8000000000 0.5272924240 1.0545848481

9 0.9000000000 0.4448580662 1.7794322649

10 1.0000000000 0.3678794412 0.3678794412

------------------------------------------------------------------

*Simpson Rule* approximation ≈ 0.7468249483

***Exact Trapezoid Simpson***

------------------------------------------------------------------------

Value: 0.7468241328 0.7462107961 0.7468249483

------------------------------------------------------------------------

Error: 0.08213% 0.00011 %

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.2000000000 0.9805806800 1.9611613600

2 0.4000000000 0.9284766900 1.8569533800

3 0.6000000000 0.8574929300 1.7149858600

4 0.8000000000 0.7808688100 1.5617376200

5 1.0000000000 0.7071067800 1.4142135600

6 1.2000000000 0.6401844000 1.2803688000

7 1.4000000000 0.5812381900 1.1624763800

8 1.6000000000 0.5299989400 1.0599978800

9 1.8000000000 0.4856429300 0.9712858600

10 2.0000000000 0.4472136000 0.4472136000

--------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 1.44303943

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.2000000000 0.9805806800 3.9223227200

2 0.4000000000 0.9284766900 1.8569533800

3 0.6000000000 0.8574929300 3.4299717200

4 0.8000000000 0.7808688100 1.5617376200

5 1.0000000000 0.7071067800 2.8284271200

6 1.2000000000 0.6401844000 1.2803688000

7 1.4000000000 0.5812381900 2.3249527600

8 1.6000000000 0.5299989400 1.0599978800

9 1.8000000000 0.4856429300 1.9425717200

10 2.0000000000 0.4472136000 0.4472136000

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 1.44363449

***Exact Trapezoid Simpson***

-----------------------------------------------------------------

Value: 1.44363548 1.44303943 1.44363449

-----------------------------------------------------------------

Error: 0.0413 % 0.0001 %

-----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.8414709848 0.8414709848

1 0.0625000000 0.8581952249 1.7163904498

2 0.1250000000 0.8745438796 1.7490877592

3 0.1875000000 0.8904281963 1.7808563927

4 0.2500000000 0.9057510229 1.8115020459

5 0.3125000000 0.9204063003 1.8408126006

6 0.3750000000 0.9342785616 1.8685571232

7 0.4375000000 0.9472424468 1.8944848937

8 0.5000000000 0.9591622435 0.9591622435

------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.4519476404

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.8414709848 0.8414709848

1 0.0625000000 0.8581952249 3.4327808996

2 0.1250000000 0.8745438796 1.7490877592

3 0.1875000000 0.8904281963 3.5617127853

4 0.2500000000 0.9057510229 1.8115020459

5 0.3125000000 0.9204063003 3.6816252012

6 0.3750000000 0.9342785616 1.8685571232

7 0.4375000000 0.9472424468 3.7889697874

8 0.5000000000 0.9591622435 0.9591622435

------------------------------------------------------------------

*Simpson Rule* approximation ≈ 0.4519764340

***Exact Trapezoid Simpson***

------------------------------------------------------------------------

Value: 0.4519764600 0.4519476404 0.4519764340

------------------------------------------------------------------------

Error: 0.00638% 0.00001%

------------------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 2.0000000000 1.4426950400 1.4426950400

1 2.1000000000 1.3478227100 2.6956454200

2 2.2000000000 1.2682994000 2.5365988000

3 2.3000000000 1.2006111700 2.4012223400

4 2.4000000000 1.1422452400 2.2844904800

5 2.5000000000 1.0913566700 2.1827133400

6 2.6000000000 1.0465599400 2.0931198800

7 2.7000000000 1.0067940700 2.0135881400

8 2.8000000000 0.9712326500 1.9424653000

9 2.9000000000 0.9392222400 1.8784444800

10 3.0000000000 0.9102392300 0.9102392300

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 1.11906112

***Simpson's Rule*** Method

*n *  

0 2.0000000000 1.4426950400 1.4426950400

1 2.1000000000 1.3478227100 5.3912908400

2 2.2000000000 1.2682994000 2.5365988000

3 2.3000000000 1.2006111700 4.8024446800

4 2.4000000000 1.1422452400 2.2844904800

5 2.5000000000 1.0913566700 4.3654266800

6 2.6000000000 1.0465599400 2.0931198800

7 2.7000000000 1.0067940700 4.0271762800

8 2.8000000000 0.9712326500 1.9424653000

9 2.9000000000 0.9392222400 3.7568889600

10 3.0000000000 0.9102392300 0.9102392300

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 1.11842787

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 1.11842481 1.11906112 1.11842787

----------------------------------------------------------------

Error: 0.0569 % 0.0003 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 1.0000000000 2.7182818300 2.7182818300

1 1.2500000000 2.2255409300 4.4510818600

2 1.5000000000 1.9477340400 3.8954680800

3 1.7500000000 1.7707949500 3.5415899000

4 2.0000000000 1.6487212700 1.6487212700

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 2.03189287

***Simpson's Rule*** Method

*n *  

0 1.0000000000 2.7182818300 2.7182818300

1 1.2500000000 2.2255409300 8.9021637200

2 1.5000000000 1.9477340400 3.8954680800

3 1.7500000000 1.7707949500 7.0831798000

4 2.0000000000 1.6487212700 1.6487212700

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 2.02065122

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 2.02005862 2.03189287 2.02065122

----------------------------------------------------------------

Error: 0.5858 % 0.0293 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.6931471800 0.6931471800

1 0.1250000000 0.7575990400 1.5151980800

2 0.2500000000 0.8259394200 1.6518788400

3 0.3750000000 0.8981232600 1.7962465200

4 0.5000000000 0.9740769800 1.9481539600

5 0.6250000000 1.0537006800 2.1074013600

6 0.7500000000 1.1368710100 2.2737420200

7 0.8750000000 1.2234445800 2.4468891600

8 1.0000000000 1.3132616900 1.3132616900

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.98411993

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.6931471800 0.6931471800

1 0.1250000000 0.7575990400 3.0303961600

2 0.2500000000 0.8259394200 1.6518788400

3 0.3750000000 0.8981232600 3.5924930400

4 0.5000000000 0.9740769800 1.9481539600

5 0.6250000000 1.0537006800 4.2148027200

6 0.7500000000 1.1368710100 2.2737420200

7 0.8750000000 1.2234445800 4.8937783200

8 1.0000000000 1.3132616900 1.3132616900

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 0.98381891

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 0.98381904 0.98411993 0.98381891

----------------------------------------------------------------

Error: 0.0306 % 0.0000 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1000000000 0.0000110500 0.0000221000

2 0.2000000000 0.0003908500 0.0007817000

3 0.3000000000 0.0032801600 0.0065603200

4 0.4000000000 0.0152762800 0.0305525600

5 0.5000000000 0.0515225400 0.1030450800

6 0.6000000000 0.1416879600 0.2833759200

7 0.7000000000 0.3384514200 0.6769028400

8 0.8000000000 0.7292652500 1.4585305000

9 0.9000000000 1.4523710400 2.9047420800

10 1.0000000000 2.7182818300 2.7182818300

-------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 0.40913975

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.1000000000 0.0000110500 0.0000442000

2 0.2000000000 0.0003908500 0.0007817000

3 0.3000000000 0.0032801600 0.0131206400

4 0.4000000000 0.0152762800 0.0305525600

5 0.5000000000 0.0515225400 0.2060901600

6 0.6000000000 0.1416879600 0.2833759200

7 0.7000000000 0.3384514200 1.3538056800

8 0.8000000000 0.7292652500 1.4585305000

9 0.9000000000 1.4523710400 5.8094841600

10 1.0000000000 2.7182818300 2.7182818300

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 0.39580225

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 0.39559955 0.40913975 0.39580225

----------------------------------------------------------------

Error: 3.4227 % 0.0512 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.5000000000 0.3390050500 0.6780101000

2 1.0000000000 0.8414709800 1.6829419600

3 1.5000000000 1.2216768700 2.4433537400

4 2.0000000000 1.2859407500 2.5718815000

5 2.5000000000 0.9462675500 1.8925351000

6 3.0000000000 0.2444270200 0.4888540400

7 3.5000000000 −0.6562553300 −1.3125106600

8 4.0000000000 −1.5136049900 −1.5136049900

--------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 1.73286520

***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.0000000000 0.0000000000

1 0.5000000000 0.3390050500 1.3560202000

2 1.0000000000 0.8414709800 1.6829419600

3 1.5000000000 1.2216768700 4.8867074800

4 2.0000000000 1.2859407500 2.5718815000

5 2.5000000000 0.9462675500 3.7850702000

6 3.0000000000 0.2444270200 0.4888540400

7 3.5000000000 −0.6562553300 −2.6250213200

8 4.0000000000 −1.5136049900 −1.5136049900

---------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 1.77214151

***Exact Trapezoid Simpson***

----------------------------------------------------------------

Value: 1.76874870 1.73286520 1.77214151

----------------------------------------------------------------

Error: 2.0288 % 0.1918 %

----------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.5000000000 0.9696969700 1.9393939400

2 1.0000000000 0.5000000000 1.0000000000

3 1.5000000000 0.1163636400 0.2327272800

4 2.0000000000 0.0303030300 0.0606060600

5 2.5000000000 0.0101362100 0.0202724200

6 3.0000000000 0.0040983600 0.0040983600

--------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 1.06427452

***Simpson's Rule*** Method

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.5000000000 0.9696969700 3.8787878800

2 1.0000000000 0.5000000000 1.0000000000

3 1.5000000000 0.1163636400 0.4654545600

4 2.0000000000 0.0303030300 0.0606060600

5 2.5000000000 0.0101362100 0.0405448400

6 3.0000000000 0.0040983600 0.0040983600

-------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 1.07491528

***Exact Trapezoid Simpson***

------------------------------------------------------------

Value: 1.06587854 1.06427452 1.07491528

------------------------------------------------------------

Error: 0.150488% .84782%

------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 1.0000000000 2.7182818300 2.7182818300

1 1.3000000000 2.8225359000 5.6450718000

2 1.6000000000 3.0956452700 6.1912905400

3 1.9000000000 3.5188918100 7.0377836200

4 2.2000000000 4.1022788600 8.2045577200

5 2.5000000000 4.8729975800 9.7459951600

6 2.8000000000 5.8730881300 11.7461762600

7 3.1000000000 7.1606294500 14.3212589000

8 3.4000000000 8.8129706000 17.6259412000

9 3.7000000000 10.9317038800 21.8634077600

10 4.0000000000 13.6495375100 13.6495375100

---------------------------------------------------------------------

*Trapezoid Rule* approximation ≈ 17.81239534

***Simpson's Rule*** Method

*n *  

0 1.0000000000 2.7182818300 2.7182818300

1 1.3000000000 2.8225359000 11.2901436000

2 1.6000000000 3.0956452700 6.1912905400

3 1.9000000000 3.5188918100 14.0755672400

4 2.2000000000 4.1022788600 8.2045577200

5 2.5000000000 4.8729975800 19.4919903200

6 2.8000000000 5.8730881300 11.7461762600

7 3.1000000000 7.1606294500 28.6425178000

8 3.4000000000 8.8129706000 17.6259412000

9 3.7000000000 10.9317038800 43.7268155200

10 4.0000000000 13.6495375100 13.6495375100

---------------------------------------------------------------------

*Simpson's Rule* approximation ≈ 17.73628195

***Exact Trapezoid Simpson***

------------------------------------------------------------------

Value: 17.73575665 17.81239534 17.73628195

------------------------------------------------------------------

Error: 0.4321 % 0.0030 %

------------------------------------------------------------------

***Exercise***

Find the ***Trapezoid*** & ***Simpson’s*** Rule approximations and error: 

***Solution***

***Trapezoid Rule*** Method

*n *  

0 1.0000000000 1.0000000000 1.0000000000

1 1.1000000000 0.9090909100 1.8181818200

2 1.2000000000 0.8333333300 1.6666666600

3 1.3000000000 0.7692307700 1.5384615400

4 1.4000000000 0.7142857100 1.4285714200

5 1.5000000000 0.6666666700 1.3333333400

6 1.6000000000 0.6250000000 1.2500000000

7 1.7000000000 0.5882352900 1.1764705800

8 1.8000000000 0.5555555600 1.1111111200

9 1.9000000000 0.5263157900 1.0526315800

10 2.0000000000 0.5000000000 0.5000000000

-------------------------------------------------------------------

Trapezoid Rule approximation ≈ 0.69377140

***Simpson's Rule*** Method

*n *  

0 1.0000000000 1.0000000000 1.0000000000

1 1.1000000000 0.9090909100 3.6363636400

2 1.2000000000 0.8333333300 1.6666666600

3 1.3000000000 0.7692307700 3.0769230800

4 1.4000000000 0.7142857100 1.4285714200

5 1.5000000000 0.6666666700 2.6666666800

6 1.6000000000 0.6250000000 1.2500000000

7 1.7000000000 0.5882352900 2.3529411600

8 1.8000000000 0.5555555600 1.1111111200

9 1.9000000000 0.5263157900 2.1052631600

10 2.0000000000 0.5000000000 0.5000000000

-------------------------------------------------------------------

Simpson's Rule approximation ≈ 0.69315023

***Exact Trapezoid Simpson***

------------------------------------------------------------

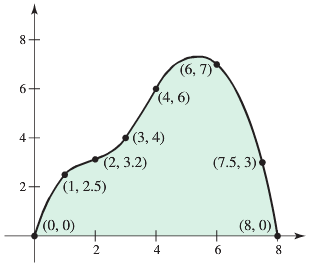
Value: 0.69314718 0.69377140 0.69315023

------------------------------------------------------------

Error: 0.0901% 0.0004%

***Exercise***

A piece of wood paneling must be cut in the shape shown below. The coordinates of several point on its curved surface are also shown (with units of inches).



1. Estimate the surface area of the paneling using the Trapezoid Rule
2. Estimate the surface area of the paneling using a left Riemann sum.
3. Could two identical pieces be cut from a 9-in by 9-in piece of wood?

***Solution***

1. The *trapezoid* Rule gives





1. The left *Riemann* sum gives



1. Although the surface area of the piece appears to be less than half of  (area of  piece of wood), the shape prohibits the creation of two identical pieces.

***Exercise***

The region bounded by the curves ,  and  is rotated about . Use Simpson’s Rule with  to estimate the volume of the resulting solid.

***Solution***

Using *Disk* method:



***Simpson's Rule*** Method

*n *  

0 0.0000000000 0.2500000000 0.2500000000

1 1.0000000000 0.5344466454 2.1377865816

2 2.0000000000 0.7758034926 1.5516069851

3 3.0000000000 0.9073974671 3.6295898684

4 4.0000000000 0.9643510838 1.9287021676

5 5.0000000000 0.9866590924 3.9466363696

6 6.0000000000 0.9950608676 1.9901217351

7 7.0000000000 0.9981787276 3.9927149105

8 8.0000000000 0.9993294122 1.9986588244

9 9.0000000000 0.9997532261 3.9990129043

10 10.000000000 0.9999092063 0.9999092063

-----------------------------------------------------------------

*Simpson Rule* approximation ≈ 8.8082465177







***Exercise***

A pendulum with length *L* that makes a maximum angle  with the vertical. Using Newton’s Second Law it can be shown that the period *T* (the time for one complete swing) is given by



Where  and *g* is the acceleration due to gravity. If  and , use Simpson’s Rule with  to find the period.

***Solution***







***Simpson's Rule*** Method 

*n *  

0 0.0000000000 1.0000000000 1.0000000000

1 0.1570796327 1.0031527554 4.0126110216

2 0.3141592654 1.0124160101 2.0248320201

3 0.4712388980 1.0271895774 4.1087583096

4 0.6283185307 1.0464308046 2.0928616093

5 0.7853981634 1.0686201540 4.2744806162

6 0.9424777961 1.0917709315 2.1835418629

7 1.0995574288 1.1135333115 4.4541332459

8 1.2566370614 1.1314314233 2.2628628466

9 1.4137166941 1.1432291699 4.5729166795

10 1.5707963268 1.1473515974 1.1473515974

----------------------------------------------------------------

*Simpson Rule* approximation ≈ 1.6825506215





